MATHEMATICAL STRUCTURES, AND TIGHT CONNECTIONS BETWEEN THEM

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A central notion arising in twentieth century mathematics is that of a mathematical structure. Through identifying the essential features of some entity (such as number, space, symmetry, network) or activity (such as counting, collecting, rearranging, computing, arguing) arising in an application domain (such as engineering, physics, computer science, philosophy) and understanding it as deeply and completely as possible, we move from thinking about specific entities or activities to thinking about more abstract entities or activities. In this way the notion of a mathematical structure evolves from entities and activities of an application domain, which may then find applications in another application domain.

While there is agreement that everything is a mathematical structure, there is a diversity of mathematical opinions of an appropriate formalism of a mathematical structure.

David Hilbert, in "Uber das Unendliche, Mathematische Annalen (95): 161-190 (1926) said:

"Noone will drive us from the paradise which Cantor created for us.". Some `attacked' set theory. For example, Wittgenstein replied

"If one person can see it as a paradise of mathematicians, why should not another see it as a joke?".

Others like Mac Lane felt that this

"is a mistakenly one-sided view of mathematics"

and his letter *Mathematical Models: A Sketch for the Philosophy of Mathematics*, in The American Mathematical Monthly, 88 (7) (1981) p 462-472, may be interpreted as saying that

"In spite of the fundamental achievements of set theory, the perfect paradise is still to be found."

From such different opinions about how to formalize the notion of a mathematical structure, different fields of mathematics (including Algebra, Logic, Topology, Category Theory) have evolved.

Typical questions then are whether two structures (not necessarily from the same field of mathematics) are in essence the same, or really different, or whether one structure might be obtained from the other via certain natural constructions.

Establishing such tight connections contributes substantially to each field of mathematics by solving some of its shortcomings (for example, solving a problem in a given mathematical field by incorporating techniques from another mathematical field), and by providing deeper understanding of the fundamental differences between fields of mathematics, which sometimes may even lead to their unification. Moreover, different mathematical structures may provide different perspectives of the entities and activities of a given application domain, and often the interplay of these different perspectives illuminate the subject matter.

Our focus in this talk will be on exploring mathematical structures and the connections between different mathematical structures, together with their applications within and outside mathematics,